Countering Explanatory Indispensabilist Platonism

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Note. The talk will focus primarily on **§§3–4**. The submitted paper developed a broader version of the project; this handout is a more accurate guide to the presentation.

1 Introduction

1.1 Explanatory Indispensability Argument (EIA)

> EIA1. We ought rationally to believe in the existence of any entity that plays an indispensable *explanatory* role in our best scientific theories.
>
> EIA2. Mathematical objects play an indispensable *explanatory* role in science.
>
> EIA3. We ought rationally to believe in the existence of mathematical objects.

Baker, 2005; A recent call for revival of EIA, see Paseau, forthcoming.

1.2 Targeted Explanatory Indispensabilist Platonism (EIP):

EIP1. There are *objects* being quantified over in apparently true mathematical sentences from the abstract domain.

EIP2. The objectualist commitment is justified by the EIA, via the alleged indispensability of mathematical objects in Distinctively Mathematical Explanations (DMEs).

EIP1: So other features viewed as building (other brands of) platonism won't be focused here, e.g., the mind-independence and absoluteness of platonic objects.

1.3 Distinctively Mathematical Explanations (DMEs)

The periodical cicada case: Why 13- and 17-year life cycles rather than nearby alternatives? Given ecological constraints in the 12–18 year range and periodic predators, prime-numbered cycles minimize overlap with predator cycles:

$$\text{primality of } 13, 17 \Rightarrow \text{fewer common multiples} \Rightarrow \text{less predator overlap.}$$

1.4 Countering Explanatory Indispensabilist Platonism (Thesis)

If counterfactual reasoning is central to the assessment of DMEs, then EIP becomes unstable: the very machinery needed to articulate explanations shifts priority away from abstract objects and toward fact-profiles, structural roles, and controlled mathematical variation.

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2 Motivations & Approaches

2.1 Counterfactuals for Mathematical Explanations

Baron, Colyvan, Ripley (BCR) (2017; 2020); see also Povich, 2024.

A common recipe for counterfactual accounts of DMEs:

> BCR1. Non-vacuous semantics: a semantics for evaluating counterpossibles (countermathematicals);
>
> BCR2. Mathematical twiddle: a policy specifying which mathematical structures are revised and allowed to have ramifications ($S \rightsquigarrow S'$);
>
> BCR3. Mapping account: an application rule $\mathfrak{F} : S \to P$, explaining how mathematical structure bears on the physical system P;

BCR1. Impossible-world (Berto and Jago, 2019) or other state-based semantics (Fine, 2021).

BCR2. Minimal relaxation (fundamentally contextual).

BCR3. $\mathfrak{F}$ may be understood as a morphism, a grounding relation or a pragmatic mapping. This connects with representation problems for DMEs (Frigg, 2022) and needs further scrutiny.

Admitted: BCR2 remains an open reconstruction problem.

1. Arbitrary revisions: Lewisian similarity principle becomes unstable for mathematical impossibilities.
2. No robust space of ramifications: it is unclear how to distinguish upstream from downstream consequences.

Kment's (2014) explanatory criterion of relevance (ECR) account aims to replace it: a shared fact counts toward closeness only if its actual explanatory history is also present in the candidate antecedent-world. The adventure of reverse mathematics and practice-oriented structuralism may also help.

2.2 Counterfactuals for Mathematical Knowledge

Example: Research on Spoof Perfect Numbers (SPN)

Summarized by Baker, 2021. SPN does occur in surveys and refereed work. Contemporary mathematicians use SPN to test upper bounds for genuine odd perfect numbers.

A spoof perfect number (SPN) is defined by means of counterarithmeticals: it would be perfect if one or more of its composite factors were prime:

If m were prime, then n would be perfect.

> If 4 were prime, then 60 would be perfect.
>
> If 22,021 were prime, then 198,585,576,189 would be perfect.

Work on spoof odd perfect numbers connects with the theory of perfect numbers itself, especially the open problem of whether any genuine odd perfect numbers exist. The case has generated structured variants:

SPN can also function as test objects: probing proof methods; testing upper bounds; exposing difficulties in the target problem; helping to generate conjectures.

- *multispoof perfect numbers*: more than one composite factor is treated as prime;
- *completely spoof perfect numbers*: all relevant composite factors are treated as prime;
- *negative spoof perfect numbers*: negative factors enter the spoof-prime profile.

There are also ordinary-looking results about SPN:

- for each fixed number of composite factors, there are finitely many even cases;
- every completely spoof perfect number has at least six composite factors;
- 907,200 is the smallest even completely spoof perfect number.

The point: the SPN case is neither empty nor arbitrary. Countermathematical reasoning can be mathematically substantive, not merely semantically exotic.

Further role in non-deductive mathematical knowledge: enumerative induction, computation, probabilistic tests, and analogy can also justify mathematical beliefs. Countermathematicals help test whether such support is sensitive, safe, and non-accidental (Paseau, 2015; Paseau, 2024). They may also help articulate truth-responsiveness in replies to epistemological challenges to platonism (Linnebo, 2006).

2.3 Modest Counterfactualism about DMEs (Approach 1)

Admitted1. More remains to be said about BCR2; structuralist resources may help specify the relevant mathematical reconstructions.

Admitted2. DMEs are not exhausted by counterfactual dependence: there are also explanations by unification, constitution, and grounding.

See: Rizzo and Schnieder, forthcoming

Claim: Counterfactual reasoning is neither expressively complete nor exclusive. It is one central instrument of explanatory practice—one among several inferential tools for articulating and assessing explanatory dependence, including where the explanans involves mathematical necessity or constraint.

Parallels Kment's (2014) treatment of counterfactuals as ingredients in theorizing modality. Compatible with modal accounts of DMEs, e.g. Lange, 2017. Roots of compatibility: counterfactuals provide epistemic frictions contributing to our modal knowledge as a conceptual reasoning machinery (Lange, 2005).

2.4 An Explanation-First Metaphysics (Approach 2)

Redirect the EIA: rather than beginning with ontological commitments and deriving explanatory schemes accordingly, we should start from an epistemological account of distinctively mathematical explanations, and only then ask which ontological commitments, if any, are warranted.

Share Lange's (2021) reorientation. Lange begins with a modal conception of DMEs, namely "explanations by constraints." My modest counterfactualism is compatible with that account.

Modality and Explanatory Reasoning:

> The study of modal facts is important for philosophy, not because these facts are of much metaphysical interest in their own right, but largely because they provide evidence about explanatory connections.

Kment, 2014. Noted: Kment's "Explanation-First" metaphysics is somewhat different: explanation plays a more fundamental role, even grounding his conception of counterpossibles. The shared guiding thought is that modal talk should be assessed by its explanatory role.

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3 Against the Platonist Face-Value Semantics (`Argument 1`)

3.1 The Platonist Semantic Appearance

An accepted advantage of platonism over nominalism:

> [Platonism] enjoys strong prima facie plausibility. For the language of mathematics strongly appears to have the same semantic structure as ordinary non-mathematical language....This appearance is also borne out by the standard semantic analyses proposed by linguists and semanticists.

Linnebo, 2018.

Consider: `"2 is prime."`

Since the nominalist denies that there are numbers, she cannot preserve both appearances in the most direct way:

- *error-theoretic nominalism*: keep the apparent object–predicate form, deny literal truth;
- *paraphrase nominalism*: keep literal truth, deny the apparent object–predicate form.

The platonic semantics seems to have an advantage. It preserves both:

- apparent truth-value: "2 is prime" is true;
- apparent logical form: "2" is an object; "is prime" predicates a property of it.

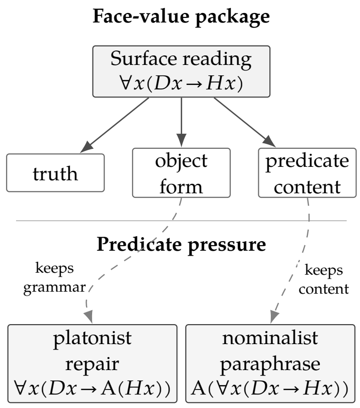
3.2 The Limits of the Face-Value Semantic Advantage

3.2.1 The Predicate-Content Pressure

But semantic appearances should also concern the *property* or *relation* apparently contributed by the predicate.

Based on Clarke-Doane, 2022

Consider: `"Every regular decagon has 10 sides of equal length."`



A = Acc: both require semantic repair

Let $D(x)$ mean "x is a regular decagon", $H(x)$ mean "x has ten sides of equal length", and $\mathrm{Acc}_{\mathrm{SG}}(\varphi)$ mean "according to synthetic geometry, φ".

Pressure: platonist wants the sentence to quantify over abstract objects satisfying $D(x)$. But the predicate $H(x)$, read in its ordinary content, is spatial (it involves sides, lengths, and relative positions), whereas abstract objects are aspatial:

- *Surface reading*:

$$\forall x(Dx \to Hx).$$

  This preserves truth-value, logical form, and predicate content. But for platonism it is too strong: H is now treated as a property literally instantiated by abstract objects.

- *Platonist repair*: keep objectual quantification, but shift the predicate content:

$$\forall x(Dx \to \mathrm{Acc}_{\mathrm{SG}}(H(x))).$$

  The abstract decagon does not literally have spatial sides; it has the property of being such that, according to synthetic geometry, it has ten equal sides.

- *Nominalist paraphrase*: keep the geometrical content, but shift the whole sentence:

$$\mathrm{Acc}_{\mathrm{SG}}(\forall x(Dx \to Hx)).$$

  No abstract decagon is required outside the theory; the surface objectual form is not taken at face value.

The point: platonism may keep objectual grammar; it does not thereby keep predicate content at face value in a way that outruns nominalism.

3.2.2 DMEs Sharpen the Pressure

The same semantic pressure reappears for DMEs.

Consider: `"Had 13 acquired 2 and 6 as factors, periodical cicadas would not have had 13-year life cycles."`

It is tempting to say that platonism handles the case more directly: the countermathematical is about the object 13. But the relevant antecedent contains a counterarithmetical predicate.

Let $S_{2,6}(x)$ mean "x has 2 and 6 as factors", P mean "periodical cicadas do not have 13-year cycles", and $\mathrm{Acc}_{\Pi}(\varphi)$ mean "according to the relevant counterarithmetical profile, φ".

Here Π is only a semantic parameter for the counterarithmetical reading. The next section asks what fixes such a profile.

- *Surface reading*:

  $$S_{2,6}(13) \Rightarrow_{\mathrm{cf}} P.$$

  This keeps objectual grammar, but it treats $S_{2,6}$ as the ordinary factor-predicate of arithmetic. That is not yet the intended counterarithmetical reading.
- *Platonist repair*: keep 13 as the object, but shift the predicate content:

  $$S^{\Pi}_{2,6}(13) \Rightarrow_{\mathrm{cf}} P, \qquad S^{\Pi}_{2,6}(x) := \mathrm{Acc}_{\Pi}(S_{2,6}(x)).$$

  The predicate is no longer simply "has 2 and 6 as factors"; it is "is such that, according to the counterarithmetical profile, it has 2 and 6 as factors".
- *Nominalist paraphrase*: keep the mathematical content under the profile-operator:

  $$\mathrm{Acc}_{\Pi}(S_{2,6}(13)) \Rightarrow_{\mathrm{cf}} P.$$

  No abstract 13 needs to stand outside the operator as the subject of predication.

Echoes the Benacerraf–Field epistemological challenge. A parallel in Linnebo's (2006) epistemological reply and my move here:

Linnebo recasts the epistemological worry as a demand for truth-responsiveness: since mathematical facts cannot vary in the straightforward way, he relocates the relevant counterfactual dependence to semantic/metasemantic variation. My move: countermathematicals also force attention away from bare objects and toward the semantic/profile machinery that makes variation evaluable.

The point: once DMEs are expressed counterfactually, platonism is not semantically simpler than nominalism. The platonist preserves objectual grammar by revising predicate content; the nominalist preserves the mathematical content by revising logical form.

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4 Against Object-First Platonism (Argument 2)

4.1 The Cross-Scenario Anchoring Problem

Cross-scenario Anchoring Condition of Counterfactual Explanation

Counterfactual explanation is supposed to track how a target would vary under a change while remaining anchored across the actual and counterfactual profiles.

> **Cross-scenario Anchoring Condition**: an explanatory counterfactual must specify what remains stable across the variation.

In ordinary interventionist cases, "same wire, lower resistance" makes clear what is varied and what is held fixed. I borrow only this anchoring requirement from Woodward: an explanatory counterfactual must say what the comparison is about across the variation. The DME question is what plays that anchoring role under a mathematical twiddle. (Woodward, 2003)

Why Platonism Struggles with Cross-Scenario Anchoring

For DMEs, the abstract object itself does not by itself anchor the relevant variation. On the arithmetical side of the cicada counterpossible, we compare the ordinary arithmetical profile S with a twiddled profile S':

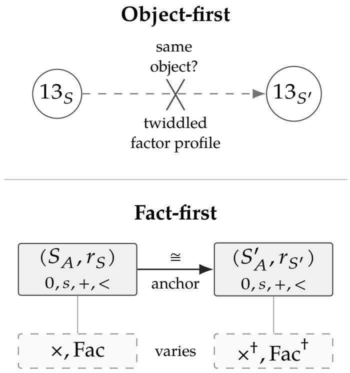
$$13_S \quad \text{vs.} \quad 13_{S'}.$$

The platonist wants these to be the very same object. But the twiddle targets 13's factor profile: 13 is supposed to have 2 and 6 as factors, and so not to be prime. This is not a change to successor, order, or addition.

- *Contingent-property reply*: Only contingent properties change; thus 13 itself remains fixed.
  Problem: for arithmetic, it is hard to say that primality and divisibility are merely accidental. On a structuralist conception, such facts help individuate the position.
- *Context-shift reply*: only the ambient structure changed; the same object appears in a different setting.
  Problem: this works only after the relevant context and embeddings are specified. Then the anchor is supplied by preserved structural relations, not by a bare object.

Fact-first answer: the whole profile is not identical across S and S'. The varied facts really vary. What remains fixed is an anchoring reduct. Let:

$$S = \langle \mathbb{N}_S, 0_S, s_S, +_S, \times_S, <_S, \mathrm{Fac}_S \rangle, \qquad S' = \langle \mathbb{N}_{S'}, 0_{S'}, s_{S'}, +_{S'}, \times^{\dagger}_{S'}, <_{S'}, \mathrm{Fac}^{\dagger}_{S'} \rangle.$$



The full profiles differ in factorhood and its immediate arithmetical consequences. A BCR-style implementation may use a local $\times^{\dagger}_{S'}$ to make the factor-twiddle evaluable without preserving ordinary multiplication wholesale. What is preserved is a smaller arithmetical reduct:

$$S_A = S{\restriction}\Sigma_A, \qquad S'_A = S'{\restriction}\Sigma_A, \qquad \Sigma_A = \{0, s, +, <\}.$$

Let $r_S = s^{13}_S(0_S)$ and $r_{S'} = s^{13}_{S'}(0_{S'})$. The anchor is not the identity of r_S and $r_{S'}$ as one object, but the preservation of their core profile:

$$(S_A, r_S) \cong (S'_A, r_{S'}).$$

Rayo's (2017) facts-first ontology gives the relevant order of explanation: do not first select objects and then ask which facts they bear; first fix the propositions/facts expressed by the relevant atomic sentences. Here, what anchors the comparison is not an object carried across scenarios, but a preserved pattern of successor/order/additive facts.

This is a profile-preservation claim, not a same-object claim. What is preserved is the successor/order/additive role of the thirteenth position. What varies lies outside this anchoring profile: factorhood, divisibility, and the downstream arithmetical facts about primality and common factors.

4.2 Against Fact-Encoded Objectualism

4.2.1 Fact-Encoded Objectualism

A stronger objectualist reply is available:

Treat mathematical objects themselves as *profile-bearing abstracta*: objects whose identity is fixed by the facts, roles, or theory-truths they encode.

In the Abstract Object Theory (`AOT`), abstract objects need not exemplify their defining properties; they encode them. Nodelman–Zalta (2014) use this to treat structures and their places as abstract objects fixed by a theory-profile; Murphy (2019) develops the idea for ante rem structuralism. This might be the strongest objectualist reply: build the profile into the object.

So the reply to the previous section is:

> The entities in countermathematicals are still objectual. We keep the same abstract object, but consider it with a different profile of encoded properties.

This object-first resistance accepts profile-sensitivity, but tries to recover object-priority by building the profile into the object.

4.2.2 Problems for Fact-Encoded Objectualism

Grant the encoding move. Once a profile has been specified, one may introduce an abstract object encoding that profile. But this comes too late to explain the counterfactual reasoning. It gives us bookkeeping, not a selection principle.

The real mathematical work is done before any objectual repackaging:

- fix what is retained: successor, order, and additive role;
- fix what is varied: the factorhood/divisibility profile;
- fix what is allowed to follow: selected consequences for primality and common-factor structure.

Encoding can then repackage this reconstruction as an object with encoded properties. But it does not tell us why this reconstruction is the right one. The explanatory question is not whether some object can be registered for the selected profile; it is why this profile is the right counterfactual reconstruction.

Example: odd perfect numbers.

> If an odd perfect number exists, it must satisfy severe structural constraints.

The point of such results is not that we first locate an odd-perfect-number object and inspect its encoded properties. We do not know whether any odd perfect number exists. Still, we can prove conditional constraints from the definitions and background theory.

For instance, any odd perfect number must have Eulerian form $p^{\alpha}m^2$, where p is prime, $p \equiv \alpha \equiv 1 \pmod 4$, and $\gcd(p, m) = 1$. The existence question remains open.

This is the case for much of practice around `DMEs`:

- introduce a hypothesis;
- derive constraints, bounds, consistency results, or contradictions;
- only then ask whether anything satisfies the profile.

So fact-encoded objectualism can preserve object-talk, but only by encoding the very profile that already does the explanatory work.

If object-talk is retained at all, the conception of objecthood should be closer to Rayo's (2025) ultra-thin/network objecthood: objects are nodes made salient by a network of connections, not occupants of a prior metaphysical "bucket". The object is licensed by the profile; it does not ground the profile.

The point: controlled structural consequence has priority over assigning objects to mathematical positions. The relevant task is not to assign an object to every mathematical position, but to understand which fact-profile is coherent, what it entails, and how it supports the explanation.

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5 Conclusions

5.1 Fact-First Ontology & `DMEs`

1. Mathematical entities referred to in `DMEs` should be understood, in the first instance, as structures or facts rather than as bare abstract objects.
2. Countermathematicals should be understood as a way of navigating the space of possible structures. By means of proofs, models, consistency results, and independence results, we chart a map of what would follow if things were thus and so.
3. The indispensability of mathematics in `DMEs` does not stem from the discovery of *abstract truthmakers*, but from the indispensable role mathematical structures play as *inferential tools*. The point of mathematics in sciences may be understood as helping us organize logical space and acquire inferential templates that can be deployed across contexts.

The structuralist background is Feferman's (2014) conceptual structuralism: mathematical objectivity lies in the stability and coherence of shared structural conceptions, not in a heavyweight realm of abstract objects. In applied contexts, this becomes an inferential reading of mathematics: structures supply reusable patterns of reasoning across cases (Bueno and French, 2018).

5.2 Explanation-First Approach to `EIA`

> The Quinean ontological question "What is there?" tacitly presupposes the Aristotelian question "On what grounds what?".

Schaffer, 2009.

Ontological commitment, if it is to be more than an arbitrary list-making exercise, requires placing entities within an explanatory hierarchy, specifying the grounding or dependence relations that structure reality.

Without this, our commitments risk being disconnected from the very explanatory roles invoked to justify them. Hence, while the Quinean approach derives ontological commitment from observation and theory choice, guided by holistic confirmation, it tacitly presupposes an "Aristotelian" task: the clarification of explanatory hierarchy and grounding relations.

By shifting focus from *what* mathematics is to *how* it functions in our explanatory practices, we can reorganize the insights gained from the EIA into a practice-aligned ontology—one that secures the explanatory power of mathematics without the exorbitant metaphysical costs.

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